



Analysis of Supermirror Design

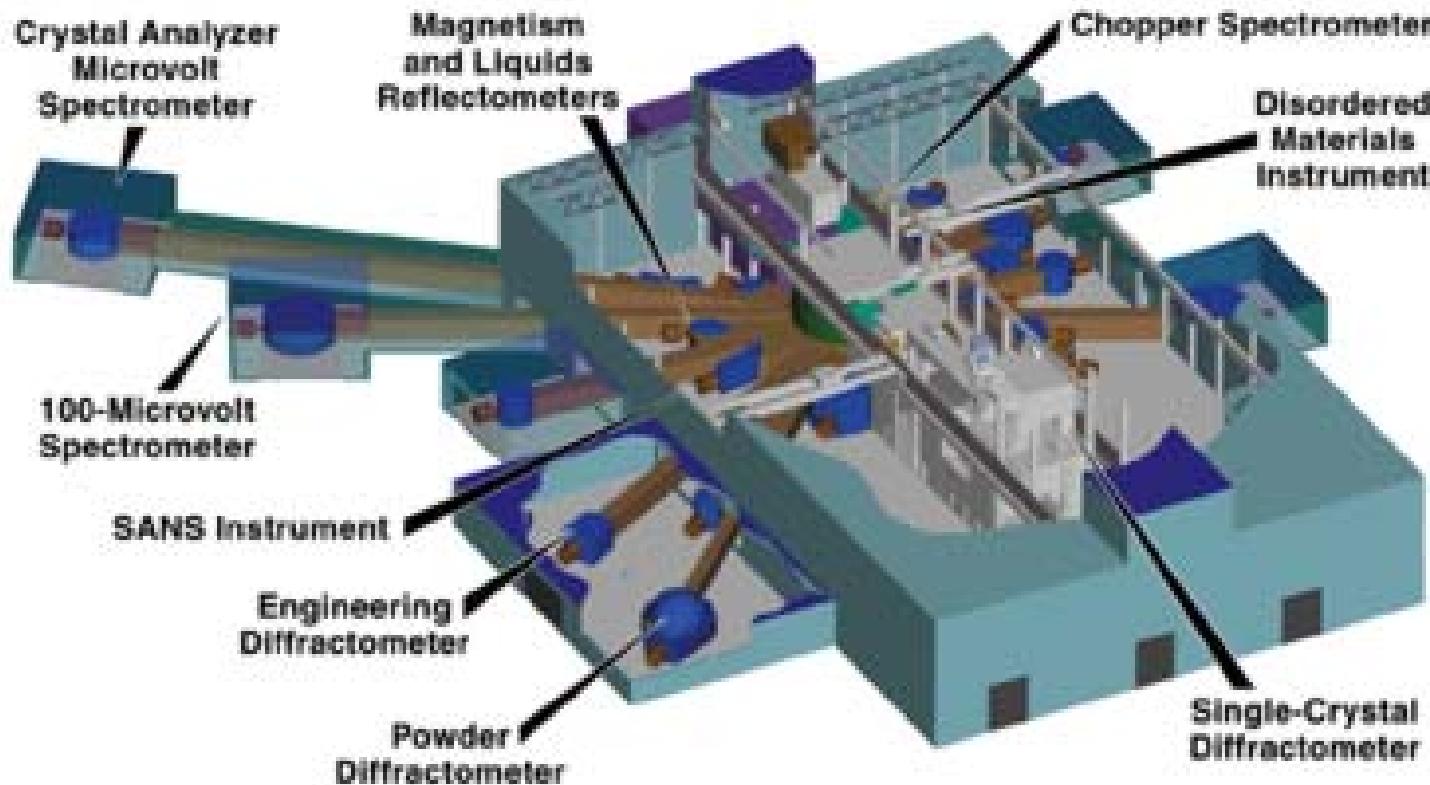
Christine Rehm

Analysis of Supermirror Design

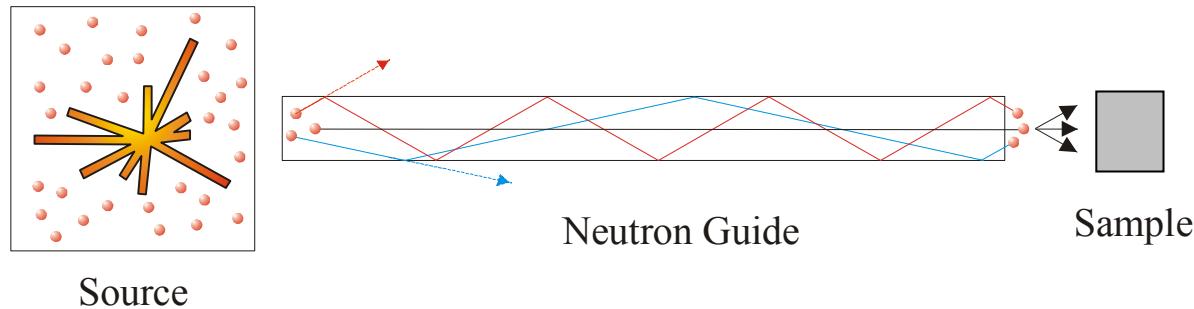


- General Introduction to Supermirrors
- Recipes for Supermirror Design
- The Influence of Various Design Parameters
- Outlook

SNS Spectrometers and Diffractometers



(Very simple) Experimental Set-up

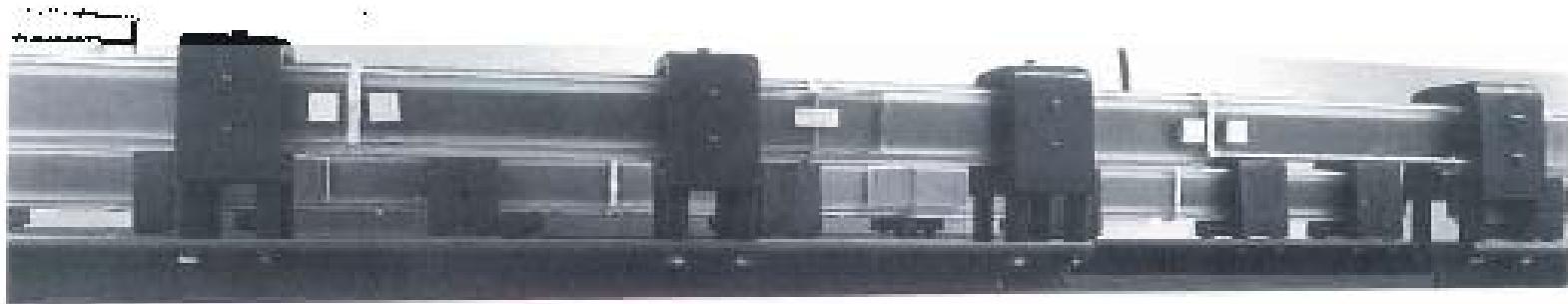


Increased flux due to increased divergence!

Glass Channel Neutron Guide

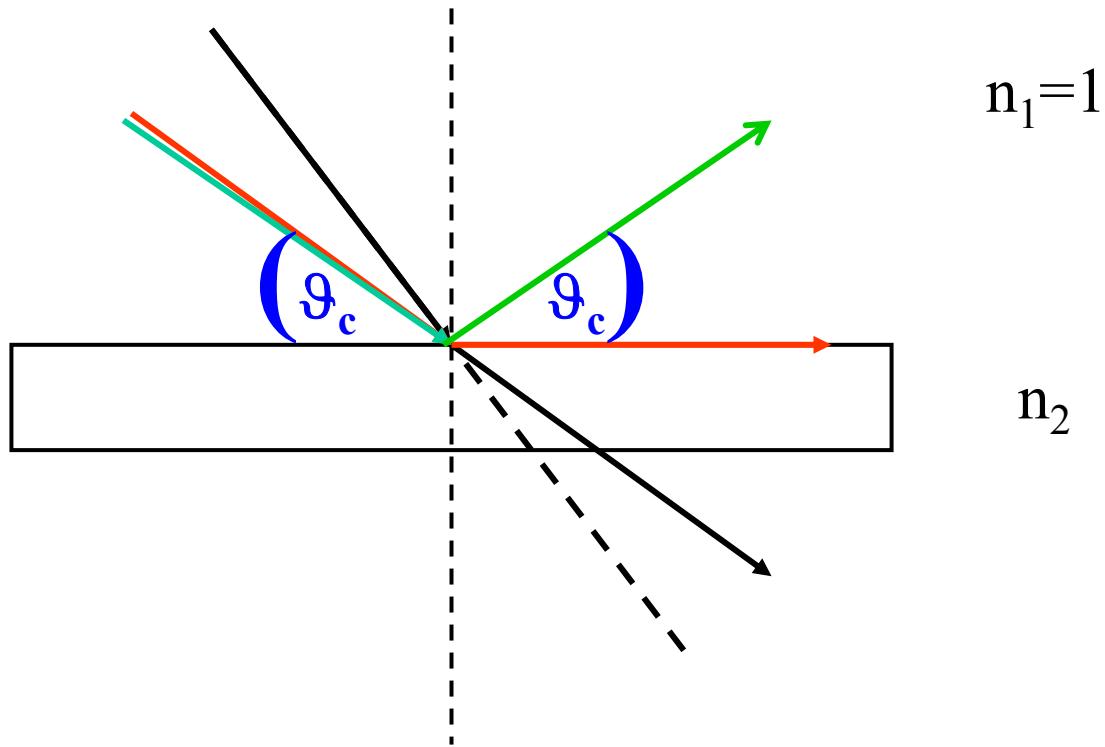


The neutron guides are up to 100 m long, assembled from ca. 0.5 m long segments fabricated to high precision and covered on their inner surfaces with a **layer with a refractive index $n < 1$, resulting in high reflectivity for neutrons**. The segments are precisely aligned and connected to one another by glass plates glued to their outer surfaces. Alignment mirrors, glued to the outside of the glass channels and precisely parallel to the inner surface, serve as reference marks during initial set-up and later checks.



Taken from the brochure
“The Spallation Neutron Source SINQ”, PSI

The Principle of Total Reflection

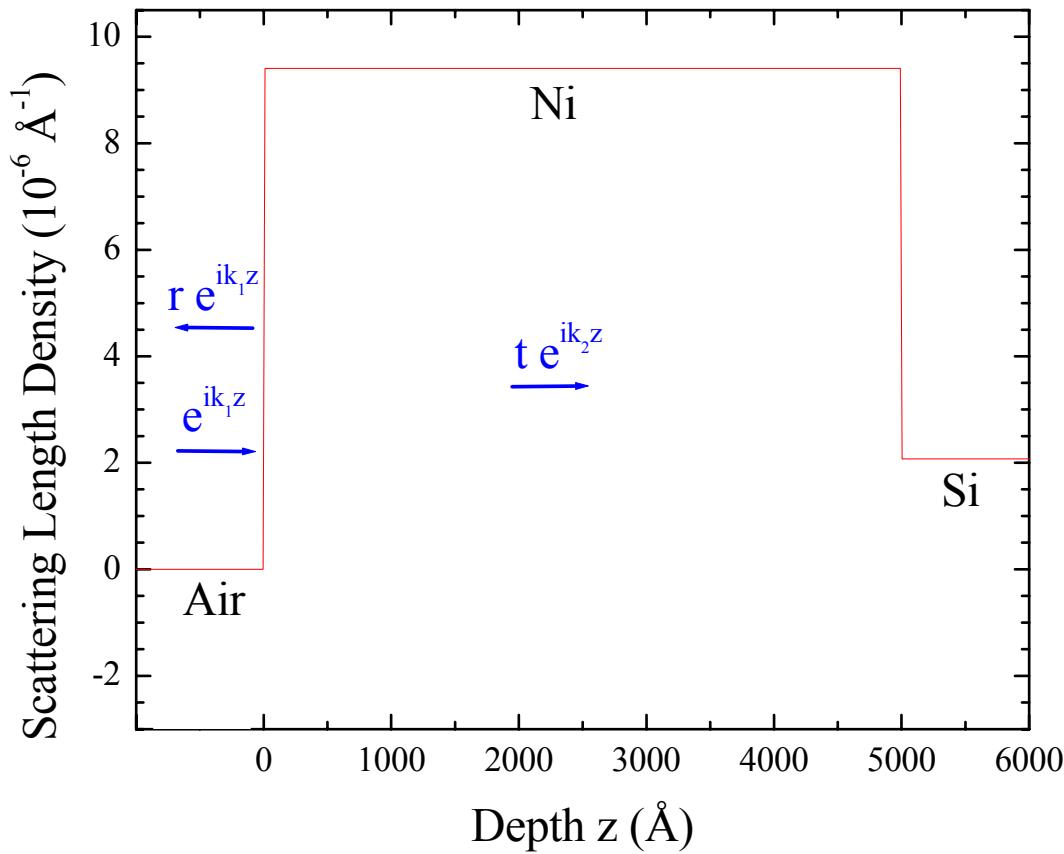


$$\vartheta_c(\lambda) = \sqrt{2[1 - n(\lambda)]} = \sqrt{\frac{Nb}{\pi}} \cdot \lambda, \quad n = 1 - \frac{\lambda^2 Nb}{2\pi}, \quad q_c = 4\sqrt{Nb\pi}$$

Theoretical Determination of Reflectivity



Si / 5000 Å Ni



Solution:

Fresnel Equations

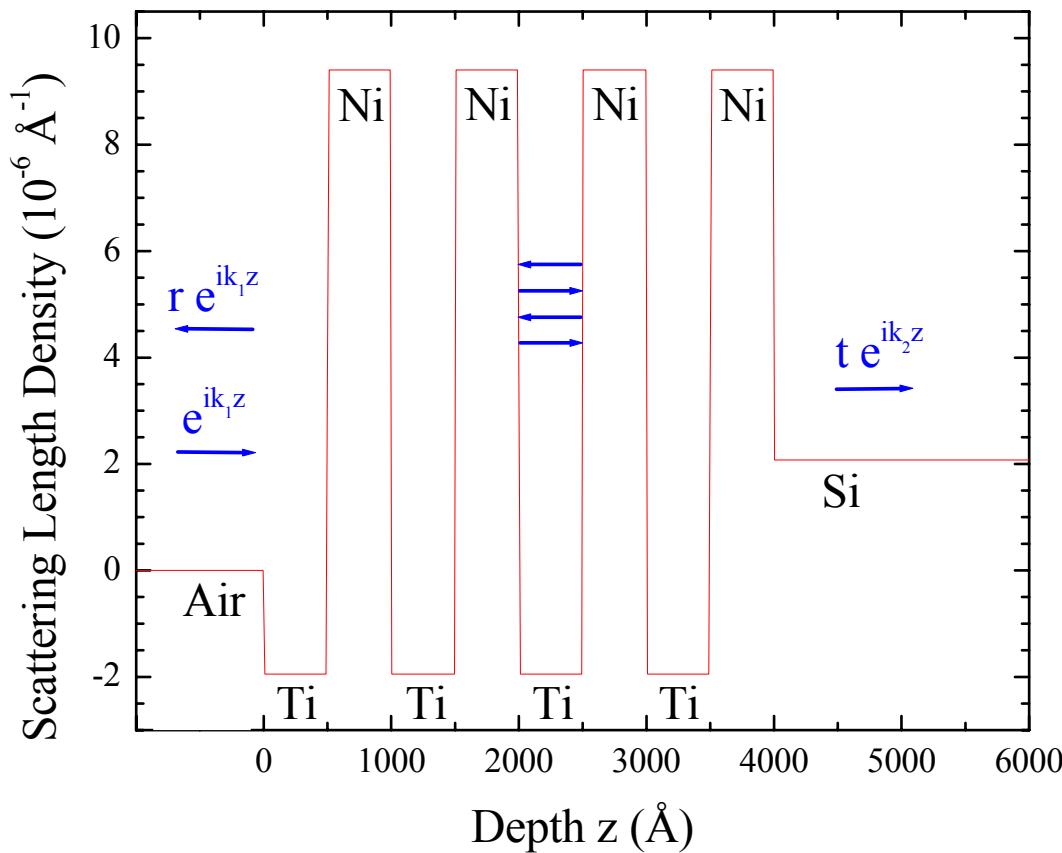
$$\text{Reflectivity } R = |r|^2 = \left| \frac{k_1 - k_2}{k_1 + k_2} \cdot e^{i2k_1 z} \right|^2$$

$$\text{Transmission } T = |t|^2 = \left| \frac{2k_1}{k_1 + k_2} \cdot e^{i2(k_1 - k_2)z} \right|^2$$

Theoretical Determination of Reflectivity



Si / [500 Å Ni / 500 Å Ti]^{*4}



Solution:

Fresnel Equations

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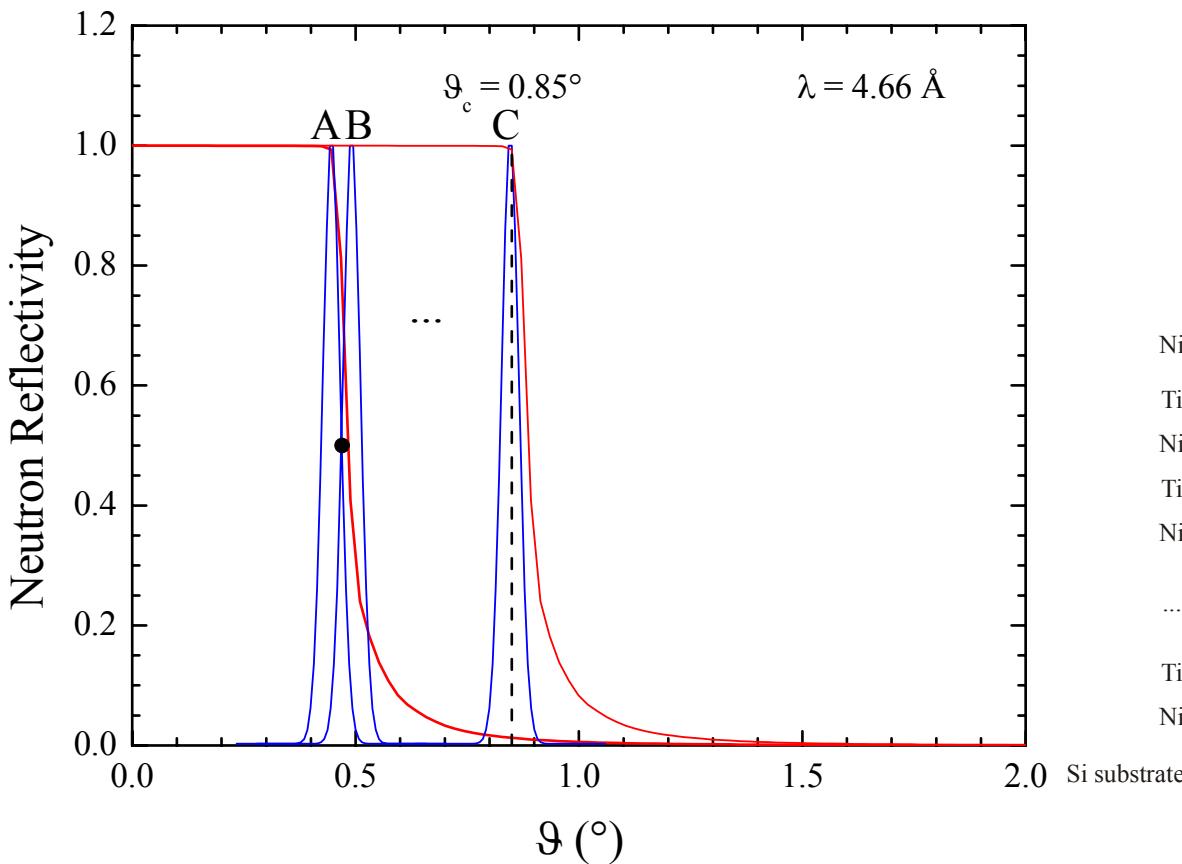
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...

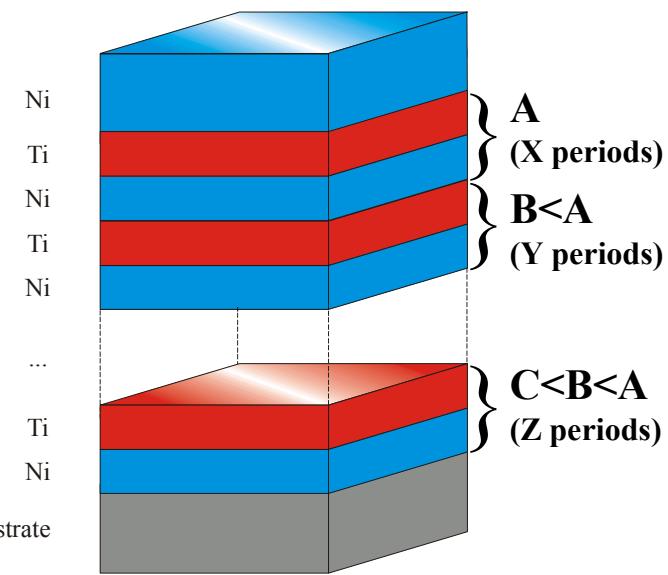
...

...

Hayter & Mook's Supermirror Design Recipe: e.g. Ni/Ti



$$(N \cdot b)^{\text{Ni}} = 9.4044 \cdot 10^{-6} \text{ \AA}^{-2}$$
$$(N \cdot b)^{\text{Ti}} = -1.9450 \cdot 10^{-6} \text{ \AA}^{-2}$$



Mezei's Supermirror Design Recipe



$$d(n) = \frac{d_c}{\sqrt[4]{n}}$$

1000 Å Ni capping layer

Ti

Ni

Ti

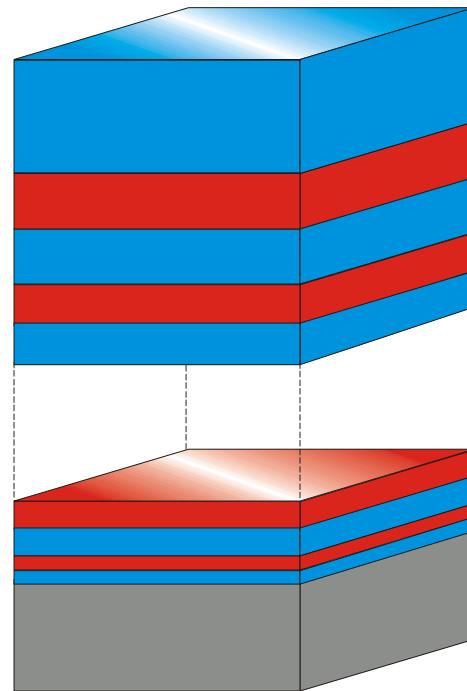
Ni

...

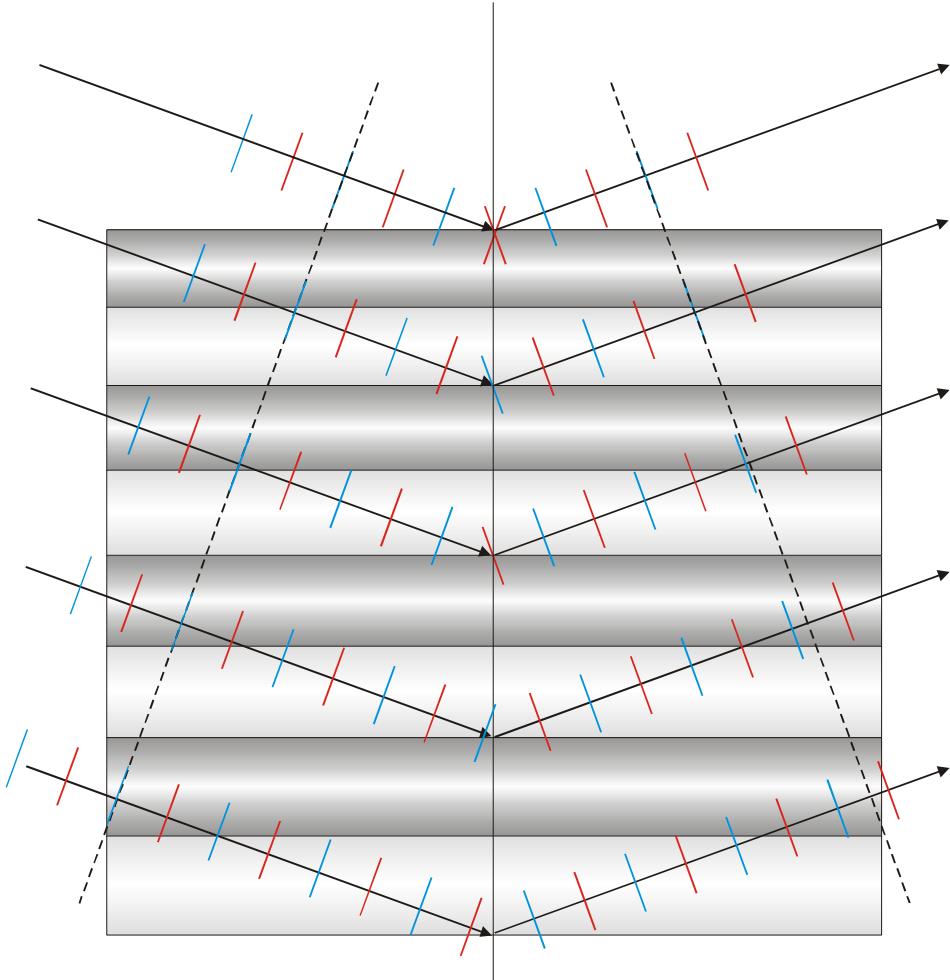
Ti

Ni

Si substrate



Mezei's Design: Limited Number of Coherently Scattering Layers



The coherence of depth-graded multilayers will not be immediately lost if the changes in bilayer thickness are not too drastic!

Mezei's Supermirror Design Recipe



Bragg's law: $2 d \sin\theta = n \lambda$

At the edge of total reflection: $\theta_c \approx \sin \theta_c$

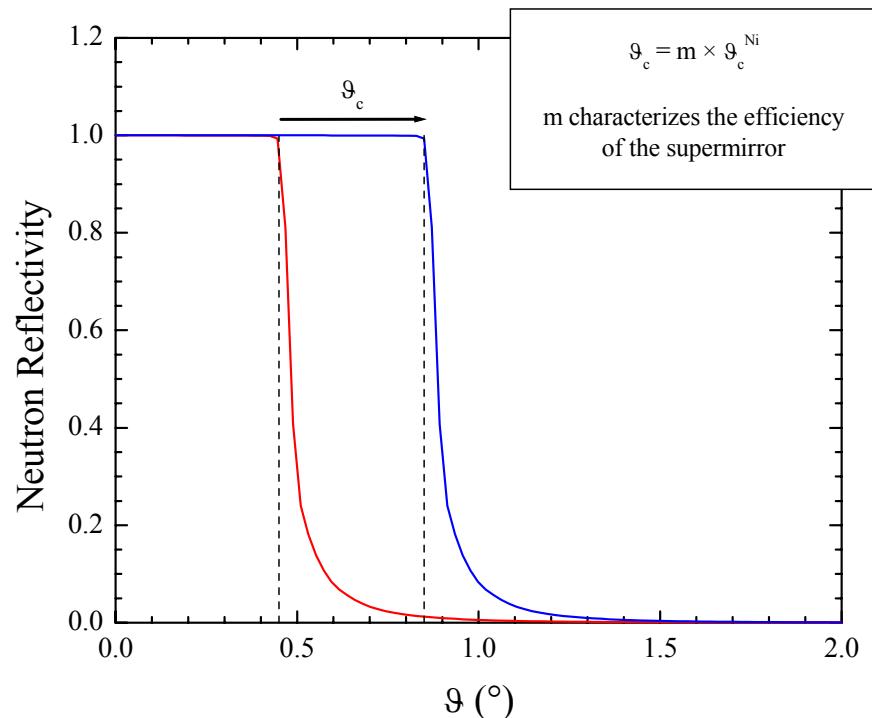
$$\Rightarrow d_c = \lambda / (2 \sin \theta_c)$$

with $\theta_c^{\text{Ni}}(\text{°}) = 0.099(\text{°}/\text{\AA}) \lambda (\text{\AA})$

$$\Rightarrow d_c^{\text{Ni}} = 290 \text{ \AA}$$

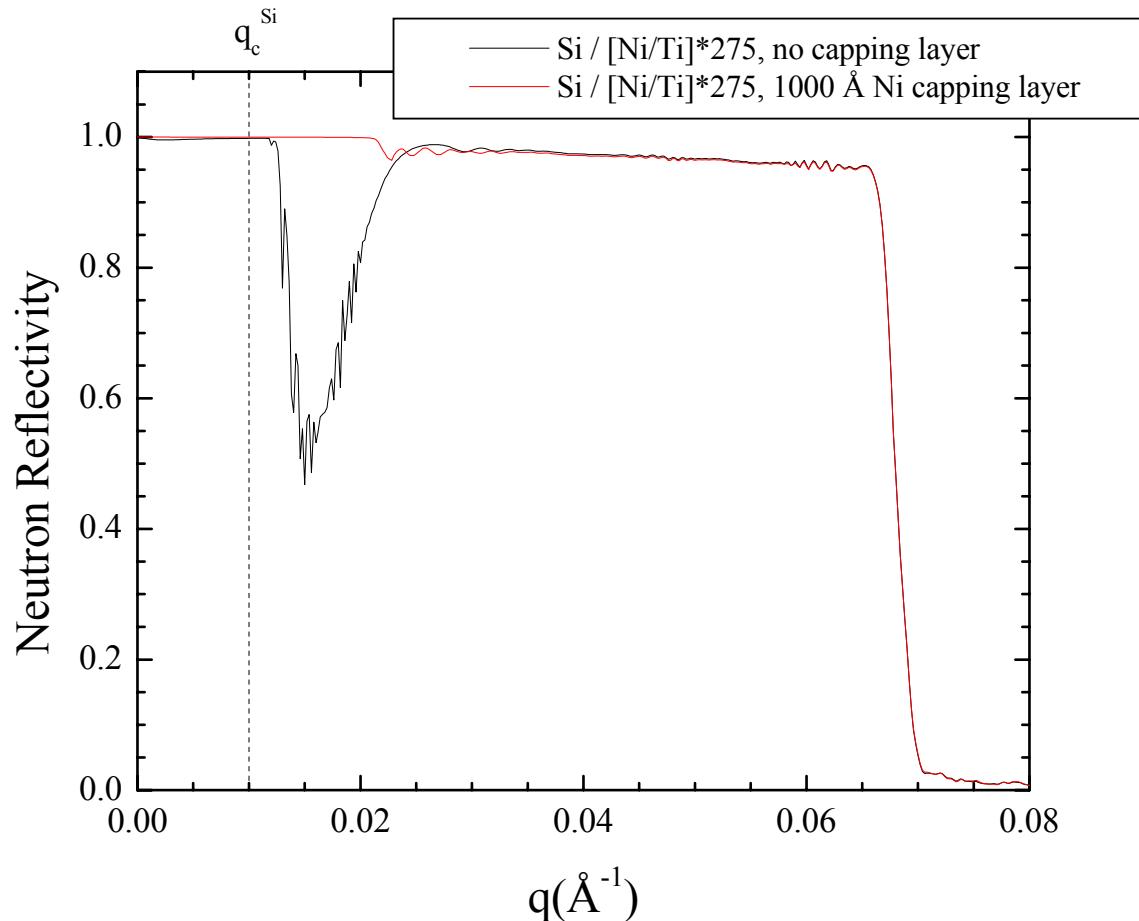
After correcting for refraction effects:

$$\underline{d_c^{\text{Ni}} = 388 \text{ \AA}}$$

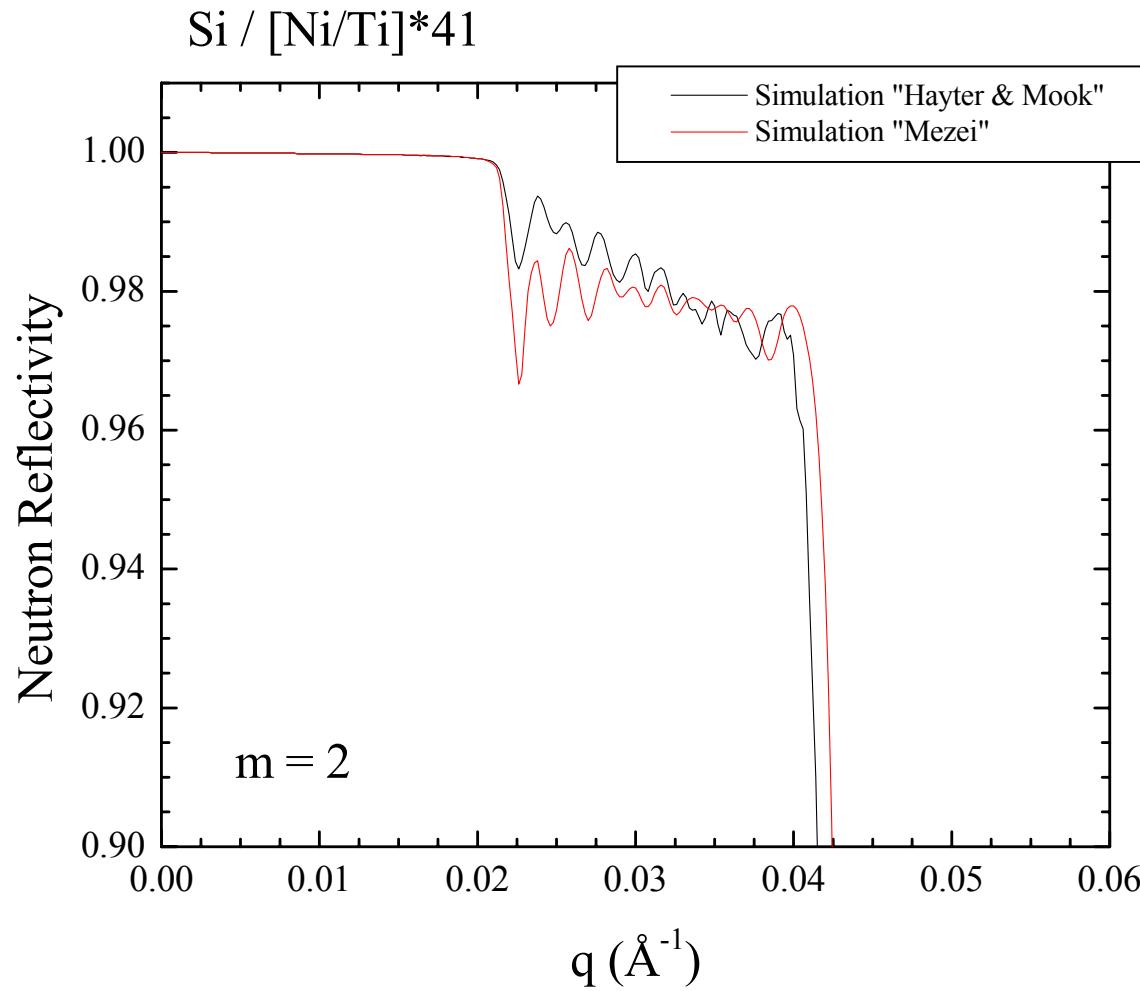


=> In order to shift the edge of total reflection to higher angles, add sequence of Bragg peaks following the edge with the first bilayer thickness being at least 388 Å thick.

Capping Layer

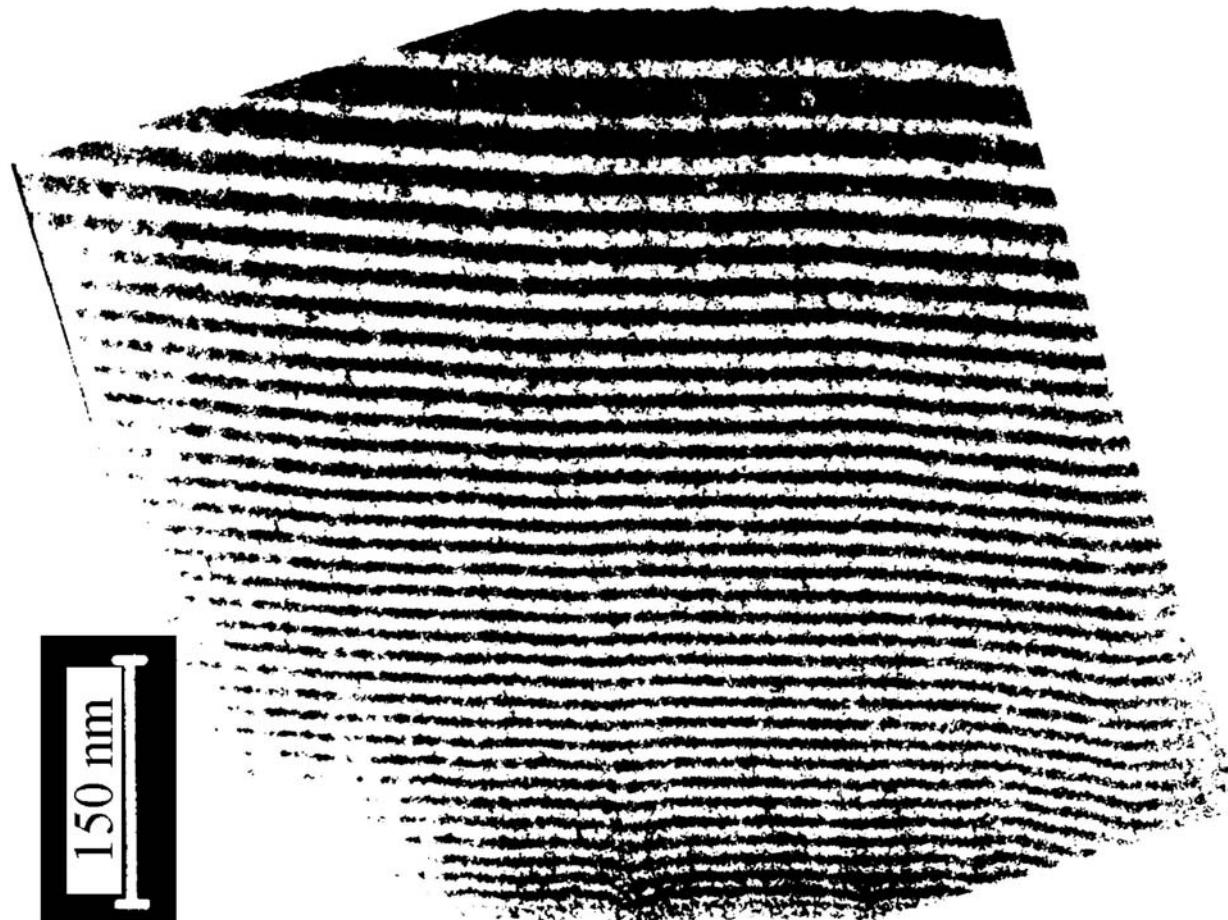


Hayter & Mook vs. Mezei



Only marginal differences !

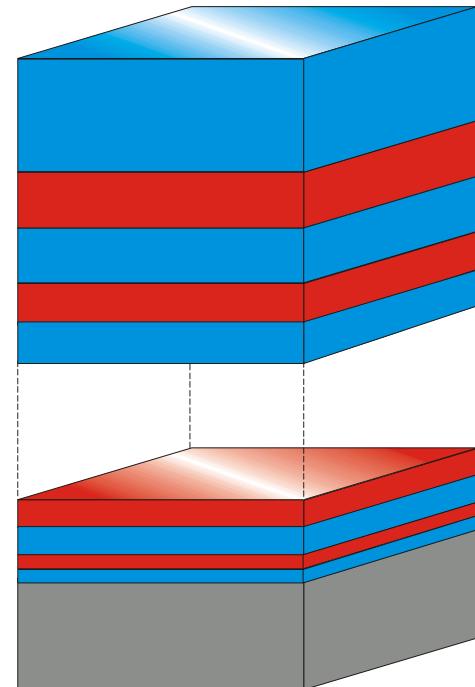
TEM Image of a Supermirror



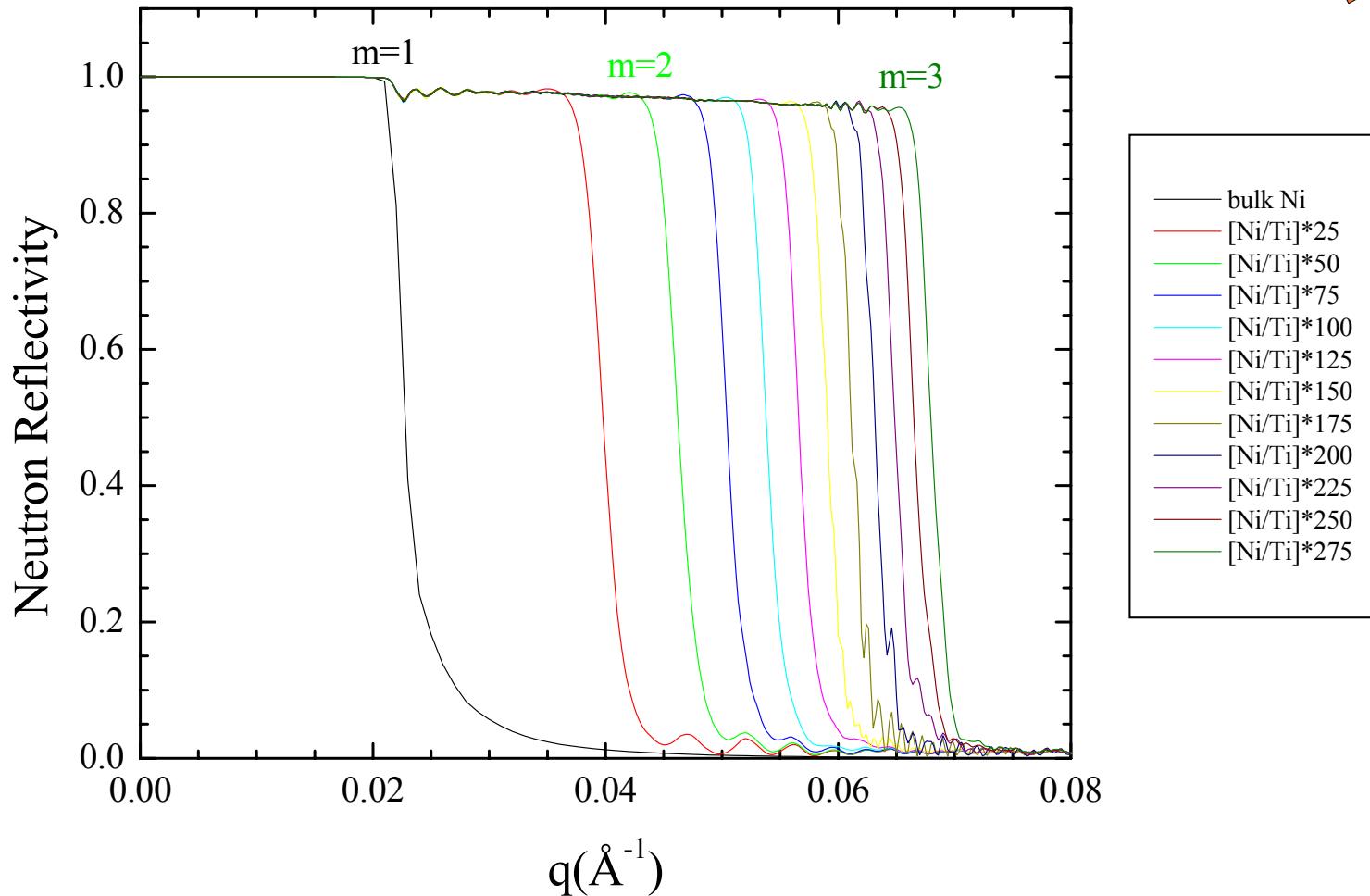
Parameters



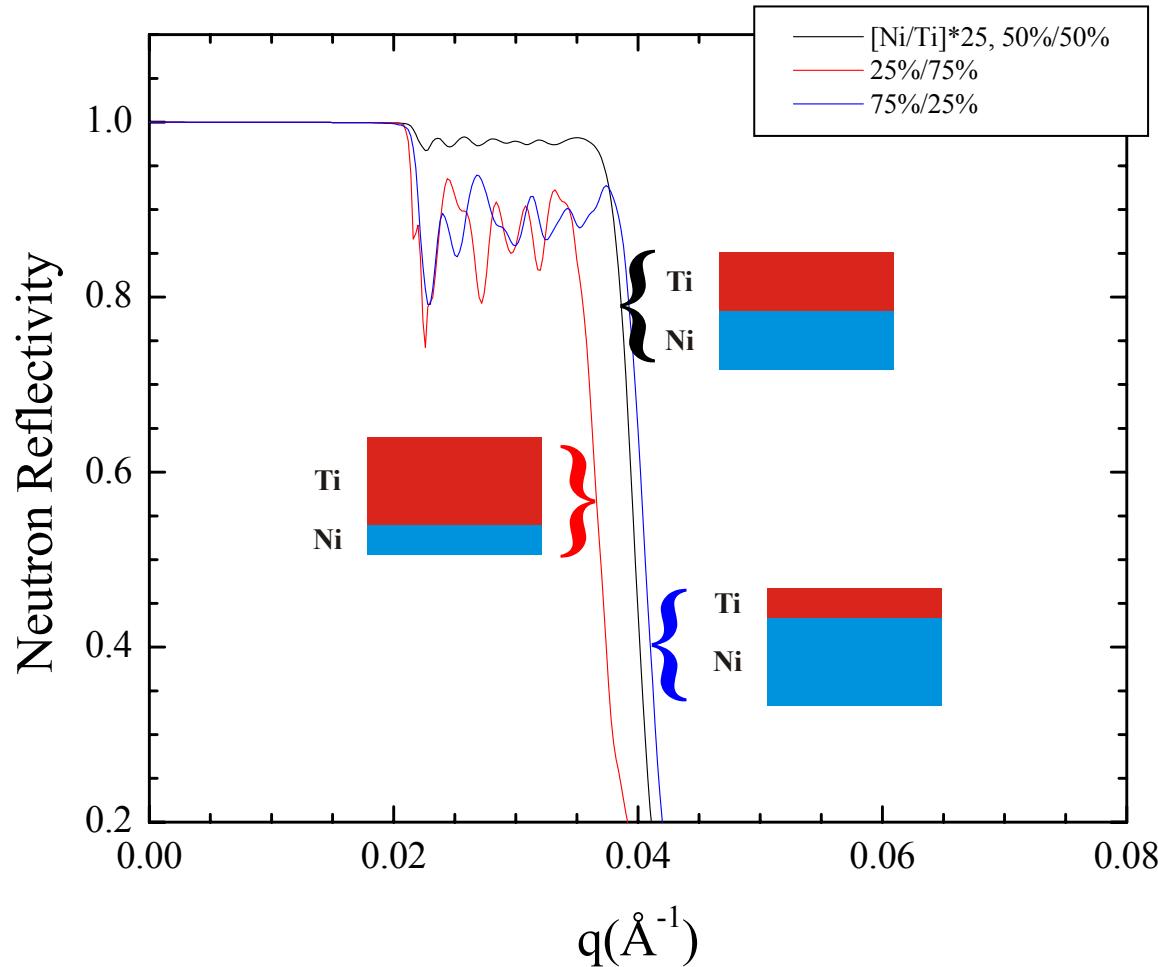
- **# of bilayers**
- **Bilayer thickness ratio**
- **Increasing/decreasing bilayer thickness**



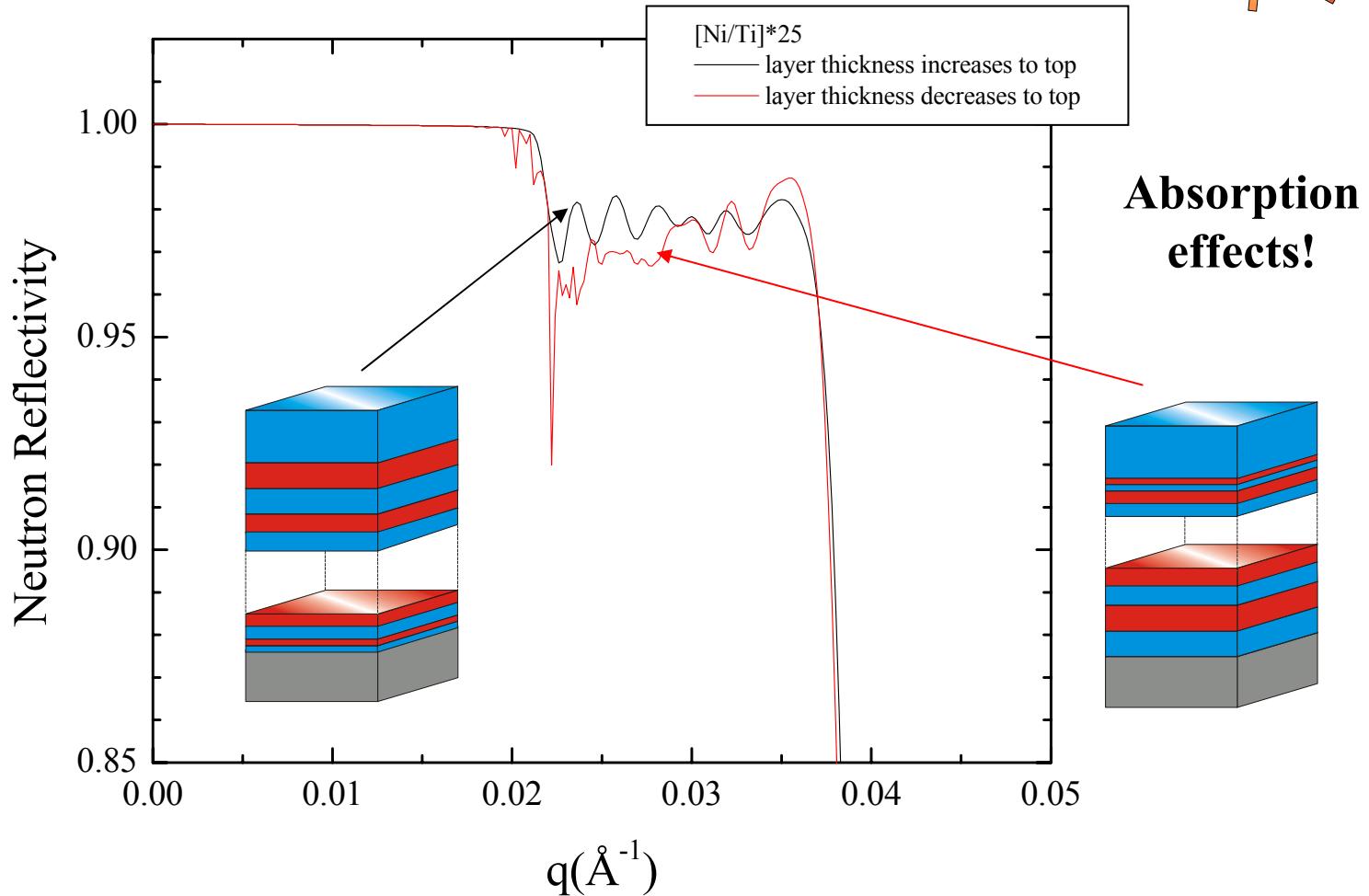
of Bilayers



Bilayer Thickness Ratio



Increasing/Decreasing Bilayer Thickness

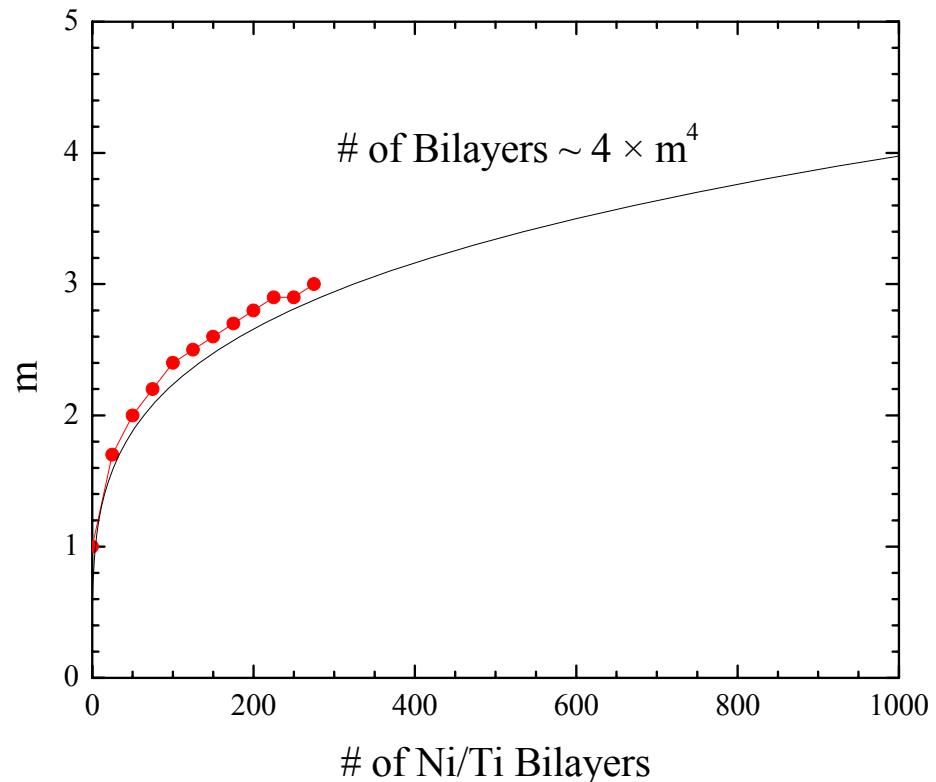


Supermirror Properties



Q: How to obtain higher m values?

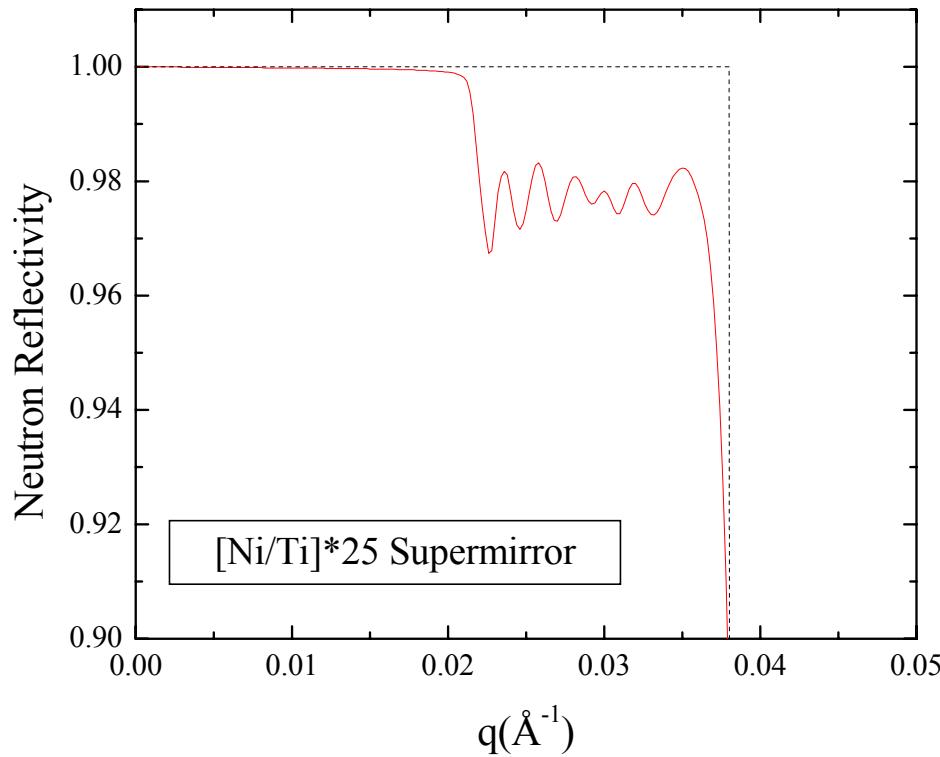
A: Add more bilayers!



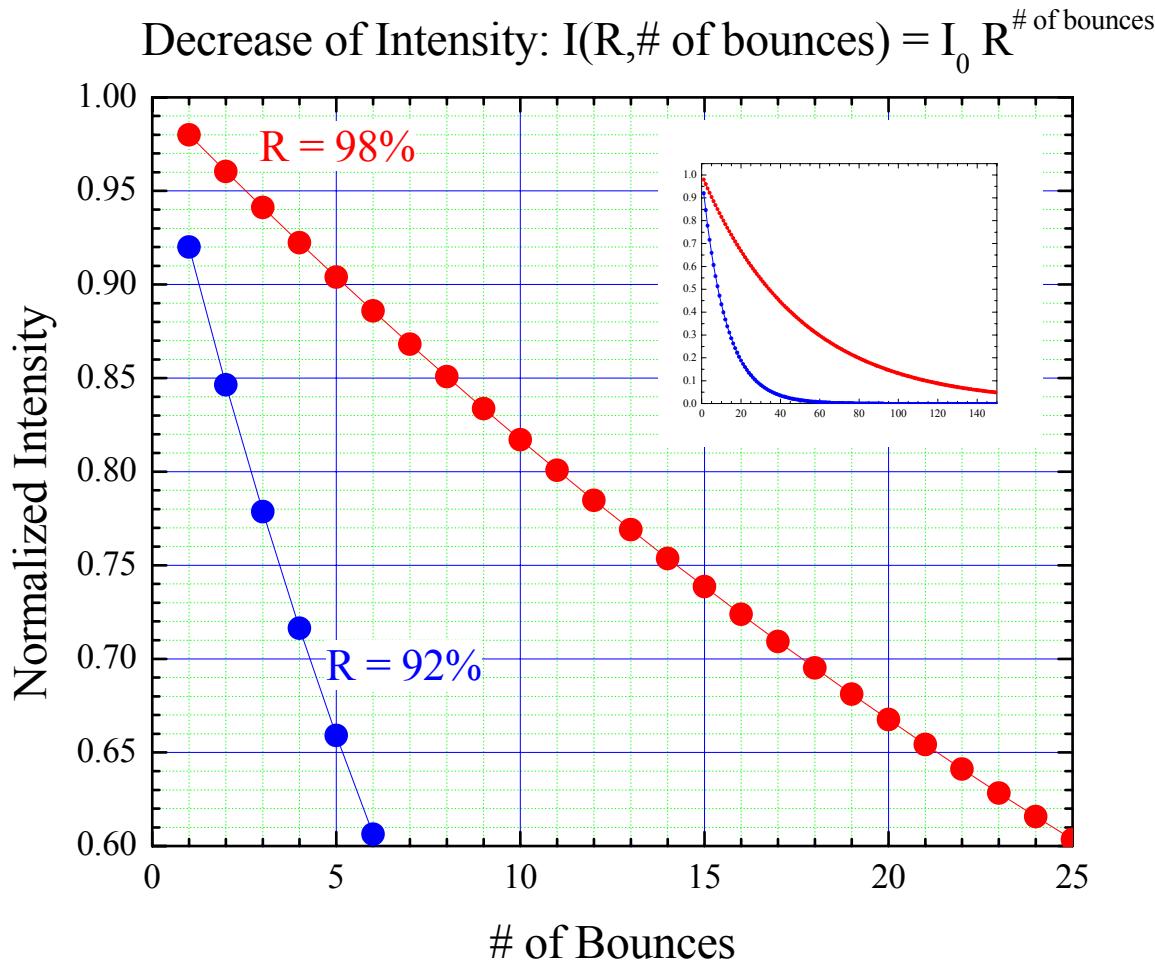
Supermirror Properties



Q: How to obtain $R = 1$ up to the edge of total reflection?



Decrease of Beam Intensity During Transport Through the Neutron Guide

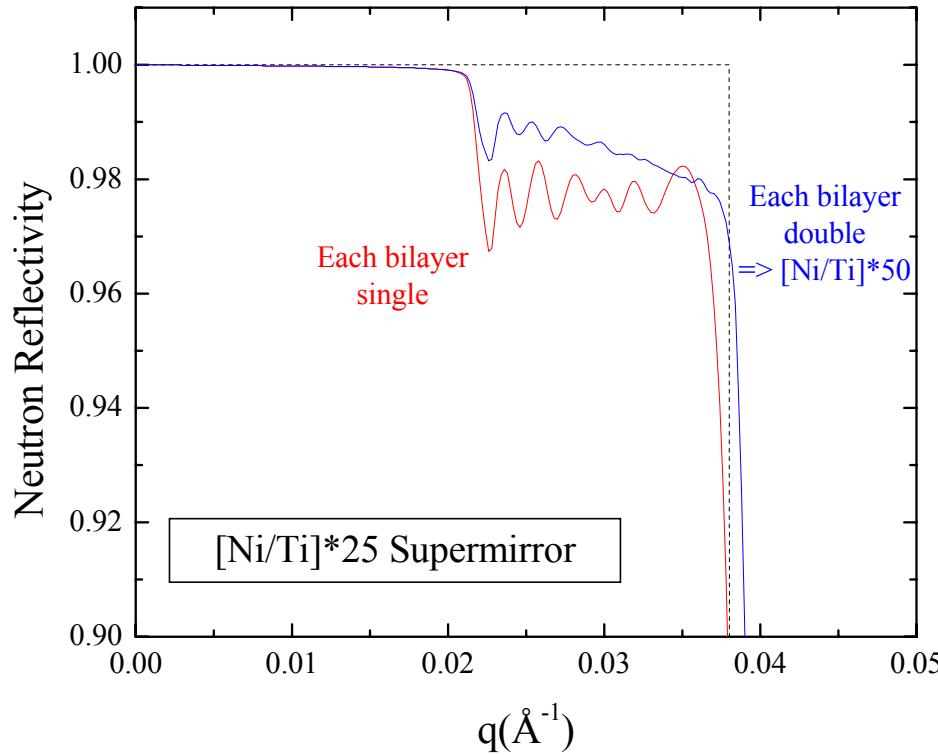


Supermirror Properties



Q: How to obtain $R = 1$ up to the edge of total reflection?

A: Change SM recipe, e.g. double each bilayer.

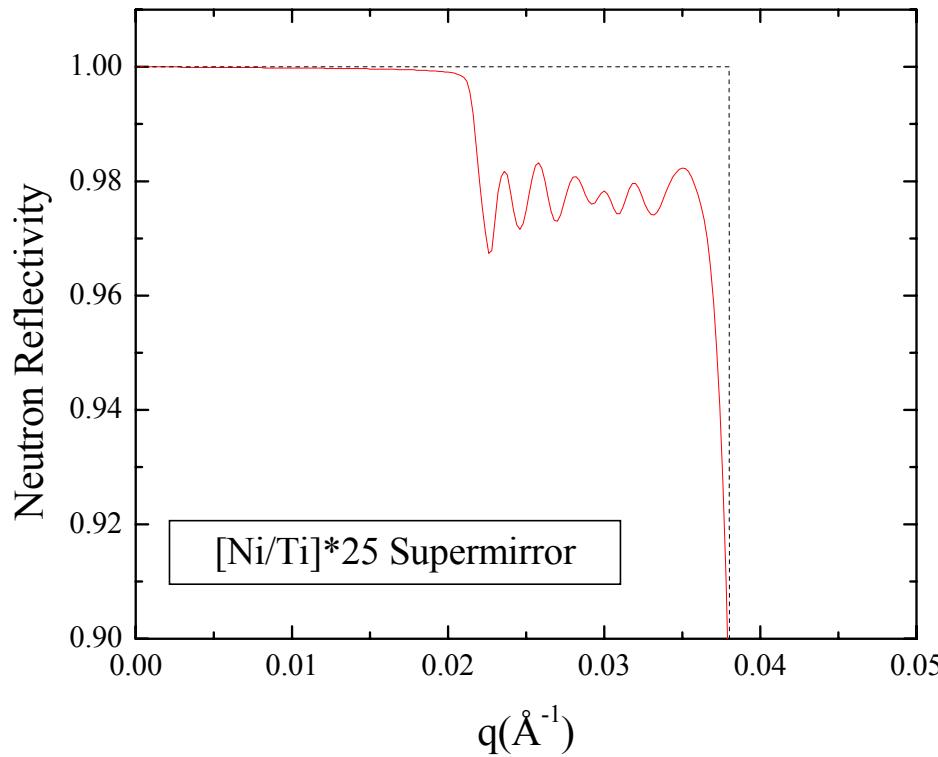


Supermirror Properties



Q: How to obtain $R = 1$ up to the edge of total reflection?

A: Increase the contrast in scattering length density of the materials.



How to increase the contrast between the supermirror materials Ni and Ti?

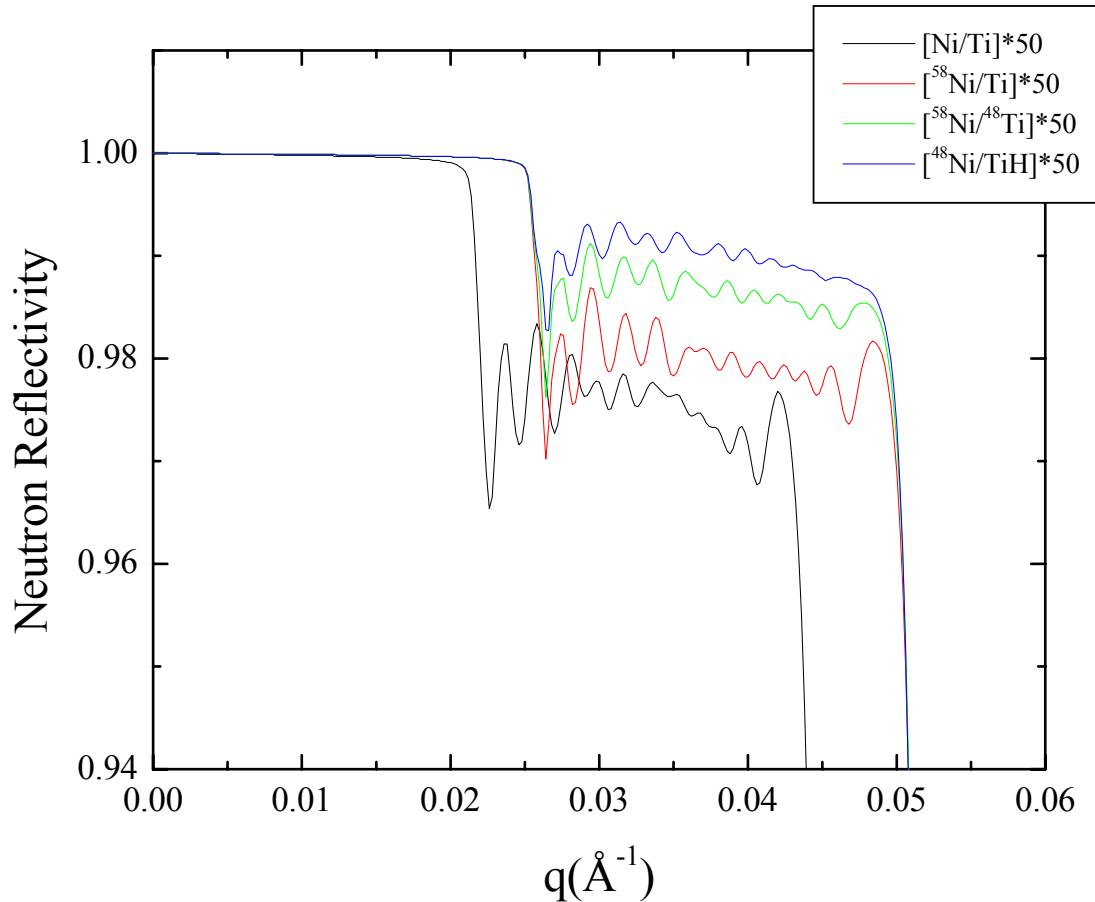


Material A	N·b (10^{-6} \AA^{-2})		Material B	$\Delta_{N\cdot b}$ (10^{-6} \AA^{-2})
Ni	9.4044	-1.945	Ti	11.3494
NiC	9.950	-1.945	Ti	11.8950
^{58}Ni	13.1479	-1.945	Ti	15.0929
^{58}Ni	13.1479	-3.4397	^{48}Ti	16.5876
^{58}Ni	13.1479	-6.0	TiH	19.1479
^{58}Ni	13.1479	-7.9435	^{62}Ni	21.0914
?	?	?	?	?

Substitution of Ni with ^{58}Ni and Ti with TiH => Higher Scattering Contrast



Material A	N·b (10^{-6} \AA^{-2})		Material B	$\Delta_{\text{N},\text{b}}$ (10^{-6} \AA^{-2})
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?	?	?	?	?

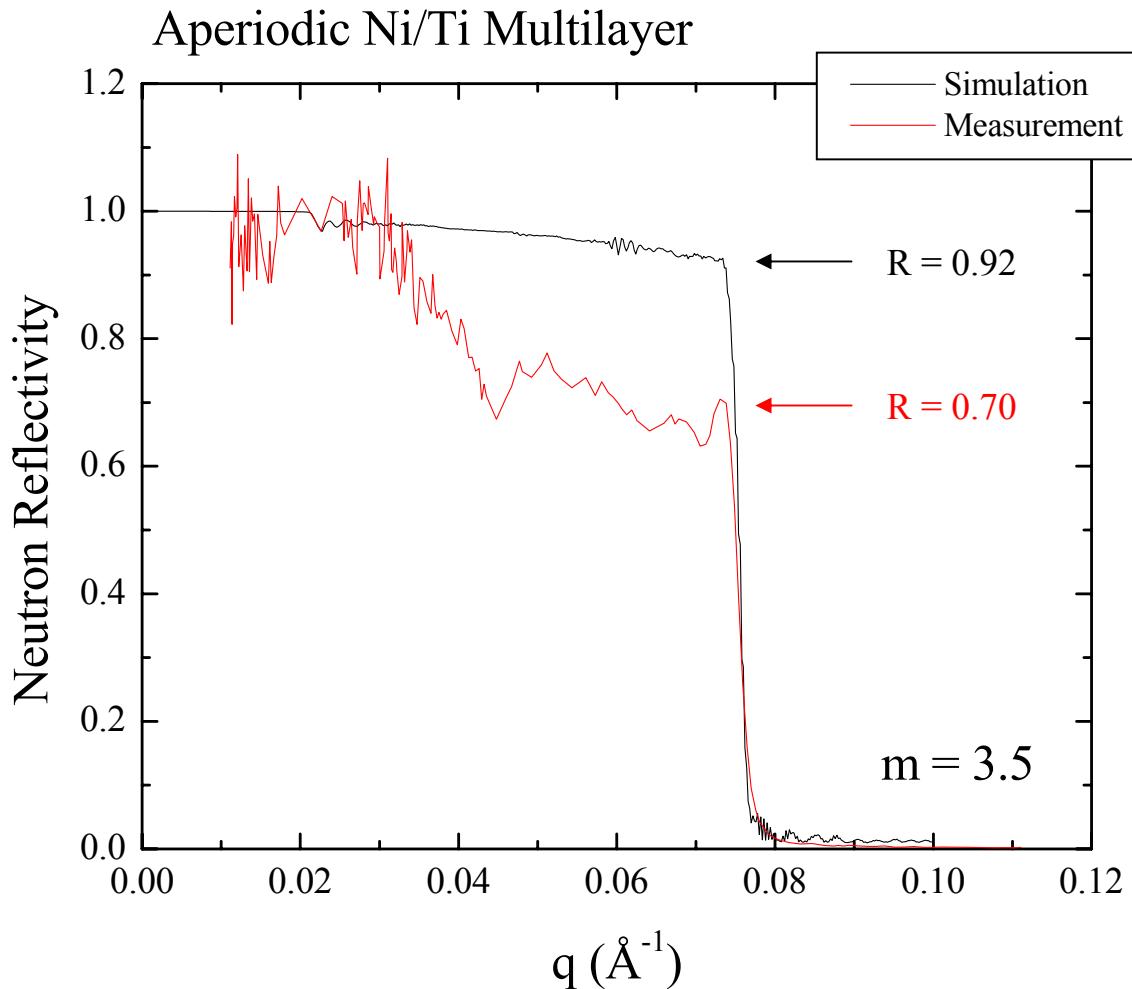


Costs Overview



Material	Quality	Material Costs per m ²	Costs for Coating per m ²
Ni/Ti	m=2	~ \$ 30	\$ 10,000
	m=3	~ \$ 100	\$ 20,000
⁵⁸ Ni/ ⁴⁸ Ti	m=2	~ \$ 10,000	\$ 20,000
	m=3	~ \$ 25,000	\$ 45,000

State-of-the-art Supermirror, PSI



Reasons for low reflectivities:

- Absorption
- Incoherent scattering
- Roughness
- Interdiffusion?
- Limited coherence introduced by thickness fluctuations?

Summary



● Supermirror Design

- Applications
- Functionality
- Recipes
- Quality improvement
- Prices for supermirror coating
- Theory vs. Reality:

Is it possible to reach the theoretical neutron reflectivity?